Aufgabe J1:

1) $X \sim N(\mu = 6\,\text{mm}, \sigma^2 = (0{,}4)^2)$

2) $P(X = 6) = 0$

3) $P(X \le a) = 0{,}85$

$$\Rightarrow P\left(\frac{x-\mu}{\sigma} \le \frac{a-\mu}{\sigma}\right) = 0{,}85$$

$$\Rightarrow P\left(Z \le \frac{a-\mu}{\sigma}\right) = 0{,}85$$

$$\Rightarrow \xi = \left(\frac{a-\mu}{\sigma}\right) \qquad \Rightarrow P(Z \le \xi) = 0{,}85$$

$$\Rightarrow \Phi(\xi) = 0{,}85$$

Aus der Tabelle: $\xi = 1{,}03 \quad \Leftrightarrow \quad \frac{a-\mu}{\sigma} = 1{,}03$

$$a = 1{,}03 \cdot 0{,}4 + 6 = \underline{\underline{6{,}412}}$$

4) $X_2 \sim N(\mu_2 = 6{,}05\ , \sigma^2 = (0{,}3)^2)$

$P(X_2 \le 6)$

$$\Rightarrow P\left(\frac{x_2-\mu}{\sigma} \le \frac{6-\mu}{\sigma}\right) = P\left(Z \le \frac{6-6{,}05}{0{,}3}\right)$$

$$\Rightarrow P\left(Z \le \frac{-0{,}05}{0{,}3}\right) = P(Z \le -0{,}16)$$

$$\Rightarrow \Phi(-0{,}16) = 1 - \Phi(0{,}16)$$

$$= 1 - 0{,}5636$$

$$= \underline{\underline{0{,}4364}}$$

5) $Y = X_2 - X_1$

$Y \sim N(\mu_2 - \mu_1, \sigma_2^2 + \sigma_1^2)$

$Y \sim N(0{,}05\ ,\ 0{,}25)$

6) Das Bohrloch ist größer als der der Stift, der Stift passt also in das Bohrloch.

7)
$$P(y < 0) = P\left(\frac{y-\mu}{\sigma} \le \frac{0-\mu}{\sigma}\right)$$

$$= P\left(Z \le \frac{-0{,}05}{0{,}5}\right) = P(Z \le -0{,}1)$$

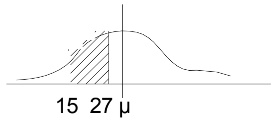
$$= \Phi(-0{,}1) = 1 - \Phi(0{,}1)$$

$$= \underline{\underline{0{,}402}}$$

Aufgabe J2:

$$X \sim N(\mu_x = 30, \sigma_x^2 = 3^2)$$
$$Y \sim N(\mu_y = 25, \sigma_y^2 = 4^2)$$

1)



$$\begin{aligned}
&P(15 \le X \le 27)\\
&= P(X \le 27) - P(X \le 15)\\
&= P\left(\frac{x-\mu}{\sigma} \le \frac{27-30}{3}\right) - P\left(\frac{x-\mu}{\sigma} \le \frac{15-30}{3}\right)\\
&= P(Z \le 1) - P(Z \le 5)\\
&\Rightarrow \Phi(-1) - \Phi(-5) = [1-\Phi(1)] - [1-\Phi(5)]\\
&= \Phi(5) - \Phi(1) = 1 - 0{,}8413 = \underline{\underline{0{,}1586}}
\end{aligned}$$

2)

$$\begin{aligned}
&P(y \le a) = 0{,}8513\\
&\Rightarrow P\left(Z \le \frac{a-\mu_y}{\sigma_y}\right) = 0{,}8513\\
&\Rightarrow P(Z \le \xi) = 0{,}8513\\
&\Rightarrow \Phi(\xi) = 0{,}8513\\
&\Rightarrow 1{,}05 = \xi \;\Leftrightarrow\; \frac{a-\mu_y}{\sigma_y} = 1{,}05\\
&\Rightarrow a = 4 \cdot 1{,}05 + 35 = \underline{\underline{39{,}2}}
\end{aligned}$$

3)

$$\begin{aligned}
&P(X > 24 \cap Y > 24)\\
&= P(X > 24) \cdot P(Y > 24)\\
&= [1 - P(Z \le 24)] \cdot [1 - P(y \le 24)]\\
&= \left[1 - P\left(Z_X \le \left(\frac{24-30}{3}\right)\right)\right] \cdot \left[1 - P\left(Z_Y \le \left(\frac{24-35}{4}\right)\right)\right]\\
&= [1-\Phi(-2)] \cdot [1-\Phi(-2{,}75)]\\
&= [1-(1-\Phi(2))] \cdot [1-(1-\Phi(2{,}75))]\\
&= \Phi(2) \cdot \Phi(2{,}75) = 0{,}9772 \cdot 0{,}9970 = \underline{\underline{0{,}9743}}
\end{aligned}$$

4)

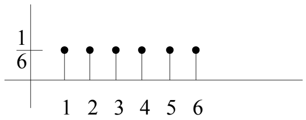
$$\begin{aligned}
&P((X-Y) < 0) = ?\\
&T = X - Y\\
&\quad \rightarrow P(T < 0)\\
&T \sim N(\mu_x - \mu_y, \sigma_x^2 + \sigma_y^2)\\
&T \sim N(-5\;,\;25)
\end{aligned}$$

$$\begin{aligned}
P(T < 0) &= P\left(Z \le \frac{0-(-5)}{5}\right) = P(Z \le 1)\\
&= \Phi(1) = \underline{\underline{0{,}8413}}
\end{aligned}$$

Zentraler Grenzwertsatz

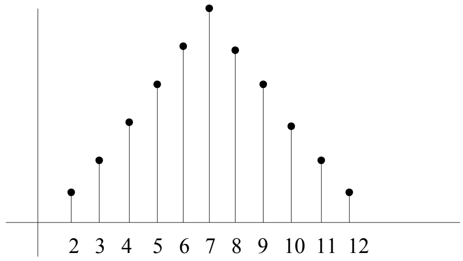
Ein Würfel: $X_1 \sim \text{gleichverteilt}\left(P(x=i)=\frac{1}{6}\right)$

i	1	2	3	4	5	6
P(X=i)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$



Zwei Würfel: $X_1 + X_2$

i	2	3	4	5	6	7	8	9	10	11	12
P(X=i)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$



Allgemein für die Verteilung der Summe:

$x_1 + x_2 + x_3 + \ldots + x_n$

- X_i beliebig verteilt $E(x_i) = \mu$
- Alle X_i i.i.d. (independently and identically distributed

$$Var(X_i)=\sigma^2$$

gilt:

$$X = X_1 + X_2 + \ldots + X_n$$
$$E(X) = E(X_1 + X_2 + \ldots + X_n) = n \cdot \mu$$
$$Var(X) = Var(X_1 + X_2 + \ldots + X_n) = n \cdot \sigma^2$$

$$X \underset{\substack{\text{approx.} \\ n \to \infty}}{\sim} N(n \cdot \mu, n \cdot \sigma^2)$$

$$\text{für } n \to \infty \quad \Rightarrow \quad n \cdot \mu \to \infty \quad ?$$
$$n \cdot \sigma^2 \to \infty \quad ?$$

$$\Rightarrow Z_n = \frac{X_1 + X_n + \ldots + X_n - n \cdot \mu}{\sqrt{n}\,\sigma}$$

$$\boxed{\Rightarrow Z_n \overset{n \to \infty}{\underset{\text{approx.}}{\sim}} N(0,1)}$$

Aufgabe B2:

X_i: „Gewicht der i-ten Person“

$X_i \sim N(\mu = 72\ ,\ \sigma^2 = 15^2)$

$$Y = \sum_{i=1}^{32} X_i$$

Annahme: X_i $i.i.d.$

$$E(Y) = E\left(\sum_{i=1}^{32} X_i\right) = \sum_{i=1}^{32} E(X_i) = 32 \cdot \mu$$

$$Var(Y) = Var\left(\sum_{i=1}^{32} X_i\right) = \sum_{i=1}^{32} Var(X_i) = 32 \cdot \sigma^2$$

$$Y \overset{\text{approx.}}{\sim} N(32 \cdot 72, 32 \cdot 15^2)$$

$$Y \overset{\text{approx.}}{\sim} N(2304\ ,\ 7200)$$

$$\begin{aligned} P(Y > 2500) &= 1 - P(Y \leq 2500) \\ &= 1 - P\left(Z \leq \frac{2500 - 2304}{\sqrt{7200}}\right) \\ &= 1 - P(Z \leq 2{,}31) = 1 - \Phi(2{,}31) \\ &= 1 - 0{,}9896 = \underline{0{,}0104} \end{aligned}$$

STATISTIK 2

Stichprobenfunktion:

Eine Funktion, die aus einer oder mehr Variablen besteht, die den Zufallsvariable $X_1 + X_2 + ... + X_n$ eine neue Zufallsvariable zuordnet.

Beispiel: Seien $X_1 + X_2 + ... + X_n$

$$X_{min} = min\{X_1, X_2, ..., X_n\}$$

$$X_{max} = max\{X_1, X_2, ..., X_n\}$$

$$\bar{X}_n = \frac{1}{n} \sum X_i$$

$$s^2 = \frac{1}{n} \sum (x_i - \bar{x})^2$$